CHAPTER 2

Current Electricity

Topic Covered

- Electric Current
- Current Density
- Drift Velocity
- Ohm's Law
- Colour Code Resistance
- Combination of Resistance
- EMF
- Kirchhoff's Laws
- Weatstone's Bridge
- Grouping of cells
- Metre Bridge
- Potentiometer
- Ammeter and Voltmeter

1. ELECTRIC CURRENT:

Electric current across an area is defined as the amount of charge flowing across the area per unit time. In metallic conductors, the current is due to the motion of electrons whereas in electrolytes and ionized gases both the ions and the electrons motion constitute current.

Consider a small area element A. If ΔQ is the charge that flows through the area in time Δt, the average current through the area is

$$I = \frac{\Delta Q}{\Delta t}$$

If rate of flow of charge is not steady then instantaneous current is given by

$$I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

The SI unit of current, Coulomb / Second, is called **Ampere (A).**

If one coulomb of charge crosses an area in one second, the current is one ampere.

Direction of current is in the direction of motion of positive charge or in the opposite direction of motion of negative charge.

PROBLEM RELATED TO ELECTRIC CURRENT:

SOLVED EXAMPLES:

Example.1 How many electrons pass through a heater wire in one minute, if current flowing is 8 ampere?

Solution. $I = \frac{q}{t} = \frac{ne}{t}$

$$n = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21}$$

Example.2 Flow of charge through a cross-section is given as $Q = 4t^2 + 2t$

(a) Find the current through the cross-section at t = 5sec.

(b) Find the average current for (0 - 10 sec)

Solution. (a) Instantaneous current

$$I = \frac{dQ}{dt} = \frac{d}{dt}\left(4t^2 + 2t\right) = 8t + 2$$

at t = 5 sec;

$$I = 8 \times 5 + 2 = 42\,\text{Amp}$$

(b) Average current

$$I = \frac{\Delta Q}{\Delta t} = \frac{Q}{t} = \frac{4 \times (10)^2 + 2 \times 10}{10} = \frac{420}{10} = 42 \text{ Amp}.$$

EXERCISE:

1. The current in a wire varies with time according to the relation

$$i = (2.0\,A) + (3.0\,A/s)t$$

How many coulombs of charge pass a cross-section of the wire in the time interval between t = 0 and t = 4.0 s?

2. Electrons in a conductor have no motion in the absence of a potential difference across it. Is this statement true or false?

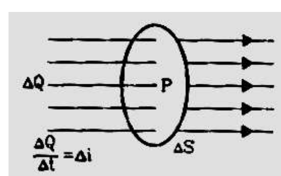
3. In an electrolyte, the positive ions move from left to right and the negative ions from right to left. Is there a net current? If yes, in what direction?

4. An electron moves in a circle of radius 10 cm with a constant speed of 4.0×10^6 m/s. Find the electric current at a point on the circle.

2. CURRENT DENSITY:

It is defined as current per unit area through any cross-section at a point in the conductor. If Δi be the current through the area Δs, at a point P perpendicular to the flow of charge, then the average current density is

$$j = \frac{\Delta i}{\Delta s}$$



The current density at the point P is

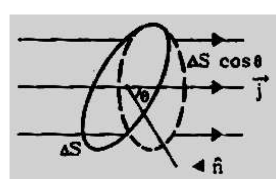
$$j = \lim_{\Delta s \to 0} \frac{\Delta i}{\Delta s} = \frac{di}{ds}$$

Current density $\vec{j}$ is a vector. It has the same direction as that of current.

If area is not perpendicular to the current

$$j = \frac{\Delta i}{\Delta s \cos\theta}$$

$$i = \int \vec{j} \cdot \overrightarrow{\Delta s}$$



PROBLEMS RELATED TO CHARGE DENSITY:

SOLVED EXAMPLES:

Example.3 One end of an aluminium wire whose diameter is 2.5 mm is welded to one end of a copper wire whose diameter is 2.0 mm. The composite wire carries a steady current of 6.25 A. What is the current density in each wire?

Solution. $J = \frac{I}{A}$

For aluminium,

$$J = \frac{6.25}{\frac{\pi}{4} \times 2.5^2 \times 10^{-6}} = \frac{4 \times 10^6}{\pi}$$

$$= 1.27 \times 10^6 \text{ A/m}^2$$

$$= 1.27 \text{ A/cm}^2$$

For copper

$$J = \frac{6.25}{\frac{\pi}{4} \times 2^2 \times 10^{-6}} = \frac{6.25 \times 10^6}{\pi}$$

$$\approx 2 \times 10^6 \text{ A/m}^2$$

$$\approx 200 \text{ A/cm}^2$$

EXERCISE:

5. Electric field (E) and current density (J) have relation.

(1) $E \propto J^{-1}$ (2) $E \propto J$ (3) $E \propto \frac{1}{J^2}$ (4) $E^2 \propto \frac{1}{J^2}$

6. When the current i is flowing through a conductor, the drift velocity is v. If 2i current is flowed through the same metal but having double the area of cross-section, then the drift velocity will be

(1) v / 4 (2) v / 2 (3) v (4) 4v

7. The current flowing through a wire depends on time as $I = 3t^2 + 2t + 5$. The charge flowing through the cross-section of the wire in time from t = 0 to t = 2 sec. is

(1) 22 C (2) 20 C (3) 18 C (4) 5 C

3. DRIFT VELOCITY:

In the absence of an electric field, the free electrons in a conductor move randomly in all directions and thereby their average velocity is zero. $\frac{u_1 + u_2 + \ldots\ldots\ldots\ldots\ldots u_n}{n} = 0$.

When an electric field is applied across the conductor, free electrons are accelerated opposite to the direction of the field however, they accelerate for a short time as these electrons frequently collide with vibrating atoms or ions or other electrons of conductor and loose energy.

The average velocity with which electrons move in the conductor under the influence of electric field is called Drift Velocity.

Magnitude of force acting on each electron due to electric field = eE.

Acceleration of each electron, $a = \frac{eE}{m}$

If τ be the average time between two succssive collision of electron, known as relaxation time, then drift velocity is given as $\mathbf{V_d = a\tau = \frac{eE}{m}\tau}$

Assuming that after each collision, an electron starts accelerating from zero velocity.

It is drift of electron, which constitute electric current.

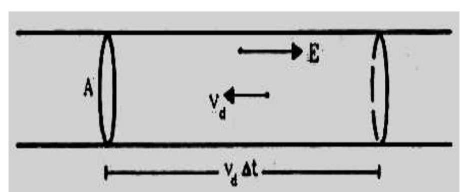
RELAXATION TIME:

Relaxation time is the average time between two successive collisions of an electron with vibrating atoms or ions or other electrons. It is constant for a given material at a given temperature.

When the temperature of a conductor is increased, the thermal agitation increases and the collisions become more frequent. Thus, the average time τ between the successive collisions (relaxation time) decreases.

Relation between current and drift velocity:

Let us consider a conductor of cross-sectional area A. Let n be the number of free electrons per unit volume. When an electric field E is applied inside the conductor, electrons move with drift velocity $V_d = \frac{eE}{m}\tau$, opposite to the electric field.



Consider a length $V_d\Delta t$ of the conductor.

Volume of this length is $AV_d\Delta t$

No. of free electrons in this length = $nAV_d\Delta t$

The charge crossing the area A in time Δt is

$$\Delta Q = nAV_d\Delta t\, e$$

$$i = \frac{\Delta Q}{\Delta t} = neAV_d$$

$$j = \frac{i}{A} = neV_d$$

PROBLEMS RELATED TO DRIFT VELOCITY:

SOLVED EXAMPLE:

Example.4 Calculate the drift velocity of electrons when 8A of current flows in a copper wire of cross sectional area $8mm^2$. (The number of free electrons of 1 cm^3 of copper is 8.5×10^{22}).

Solution. The relation between current density j and drift velocity V_d is

$$j = nev_d j \qquad \text{or} \qquad v_d = \frac{j}{ne} = \frac{i}{Ane}$$

$$= \frac{8}{(8\times10^{-6})(8.5\times10^{22}\times10^{6})(1.6\times10^{-19})} = 0.72\ mm/s$$

Example.5 Find the current flowing through a copper wire of length 0.2 m, area of cross-section 1 mm^2, when connected to a battery of 4 V. Given that electron mobility = $4.5\times10^{-6}\ m^2V^{-1}s^{-1}$ and charge on electron $= 1.6\times10^{-19}$ C. The number density of electron in copper is $8.5\times10^{28}\ m^{-3}$.

Solution. Here, $l = 0.2\,m$; A = 1 mm^2 = $10^{-6}\ m^2$; V = 4 volt ; μ = $4.5\times10^{-6}\ m^2\ V^{-1}\ s^{-1}$;

e = 1.6×10^{-19} C and n = $8.5\times10^{28}\ m^{-3}$.

We know that electric field set up across the conductor

$$E = \frac{V}{l} = \frac{4}{0.2} = 20\,Vm^{-1}$$

Now, current through wire is

$$I = nAe\upsilon_d = nAe\mu E \qquad \left[\mu = \frac{Vd}{E}\right]$$

$$= (8.5\times10^{28})\times(10^{-6})\times(1.6\times10^{-19})\times(4.5\times10^{-6})\times20 \quad = 1.2\ A.$$

Example.6 A steady current flows in a metallic conductor of non-uniform cross-section. Out of the following physical quantities, the which remains constant is

(1) current (2) current density

(3) drift speed (4) electric field

Solution. (1)

EXERCISE:

8. Drift velocity v_d varies with the intensity of electric field as per the relation

(1) $v_d \propto E$ (2) $v_d \propto \frac{1}{E}$ (3) v_d = constant (4) $v_d \propto E^2$

9. 62.5×10^{18} electrons per second are flowing through a wire of area of cross-section 0.1 m^2, the value of current flowing will be

(1) 1 A (2) 0.1 A (3) 10 A (4) 0.11 A

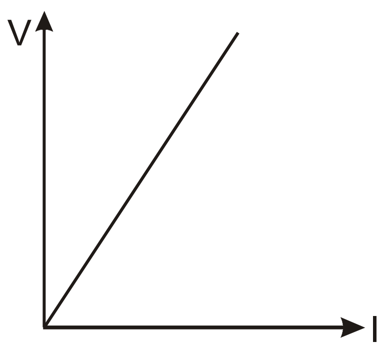
4. OHM'S LAW:

The current flowing through a conductor is proportional to the potential difference across its ends when the temperature is constant.

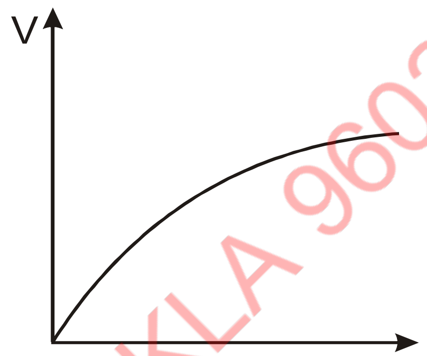
$V \propto I$ or $V = RI$

The constant of proportionality R is called the resistance of conductor. The SI unit of resistance, the volt per ampere is called an Ohm(Ω).

Ohm's law is not valid for semiconductors, electrolytes and electronic devices. These are called non-ohmic materials.



Ohmic materials



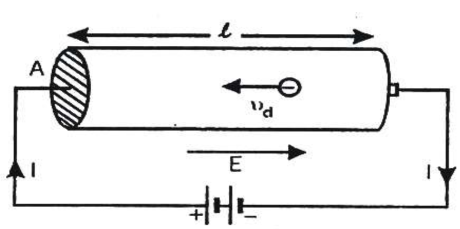
Non Ohmic materials

The resistance $R = \dfrac{\Delta V}{\Delta I}$ *is independent of current for Ohmic materials as indicated by constant slope of the line*

DEDUCTION OF OHM'S LAW:

Consider a conductor of C.S.A. A and length l. When Potential Difference 'V' is applied across the conductor, electric field inside the conductor is given as $E = \dfrac{V}{l}$.

Electrons move opposite to the electric field with drift velocity given as $V_d = \dfrac{eE}{m}\tau$



$$i = neAV_d$$

$$= neA\left(\frac{eE}{m}\tau\right) = \left(\frac{ne^2A\tau}{m}\right)E$$

$$\Rightarrow \quad i = \left(\frac{ne^2A\tau}{m}\right)\frac{V}{l}$$

$$\Rightarrow \quad V = \left(\frac{ml}{ne^2A\tau}\right)i$$

$$\Rightarrow \quad V = Ri$$

Where $R = \dfrac{ml}{ne^2A\tau}$ is the resistance of conductor and is constant for a conductor.

Resistivity or specific Resistance:

$$R = \left(\frac{m}{ne^2\tau}\right) \times \frac{l}{A}$$

$R = \rho\frac{l}{A}$, where $\rho = \frac{m}{ne^2\tau}$ is constant and is known as specific resistance or resistivity. Its unit is Ohm-metre $(\Omega - m)$

If l = 1, A = 1, R = ρ

Resistance offered by a conductor of unit length and unit area of cross-section is known as its resistivity. Resistivity of a conductor is independent of shape & size.

FACTORS ON WHICH RESISTANCE OF A MATERIAL DEPENDS:

1. Nature of the material
2. Length of the conductor, $R \propto l$
3. C.S.A of conductor, $R \propto \frac{1}{A}$
4. Temperature

Temperature dependence of Resistance:

Resistance of a conductor is given by $R = \frac{m}{ne^2\tau}.\frac{l}{A} \quad \Rightarrow \quad R \propto \frac{1}{\tau}$

When the temperature of conductor is raised, frequency of collision of free electrons with atoms / ions increases.

This reduces the relaxation time τ. Hence resistance R increass with rise in temp.

If R_0 and R_t are the values of resistance of a material of wire at 0 ºC and t ºC respectively, then over a temperature range which is not large,

$$R_t = R_0(1 + \alpha t)$$

Where α is the temperature coefficient of Resistance.

For Metals, like Silver and Copper the value of α **is positive**, hence resistance increases with rise in temperature.

For Insulators and Semiconductors, α **is negative**, hence resistance decreases with rise in temperature.

For alloys like manganin, eureka and constantan, the value of α is very small as compared to that for metals.

Hence, resistance of these alloys is almost independent of temperature. Due to high resistivity and low temperature coefficient of resistance, these alloys are used in making **standard resistance.**

Temperature dependence of Resistivity:

(i) **For metallic conductors**:

The resistivity of all metallic conductors increases with temperature.

Resistivity, $\rho = \frac{m}{ne^2\tau}$

For metals, the number of free electrons is fixed. Because of thermal agitation, the average time τ between two successive collisions decreases with the increase in temperature resulting the increase of resistivity of the material.

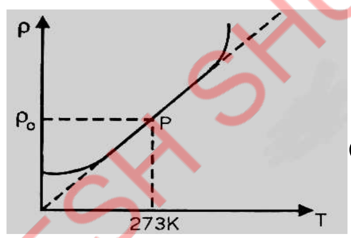
For the temperature range that is not too large, the resistivity of a metallic conductor can often be represented approximately by a linear relation

$$\rho_T = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$$

Where ρ_0 and ρ_T are the resistivity of the material at the temperature T_0 and T respectively. α is called the temperature coefficient of resistivity. This shows that temperature dependence of ρ is linear.

Some relevant points:

(a) For the elemental metals like copper and silver, the temperature dependence of ρ at low temperatures is non linear.



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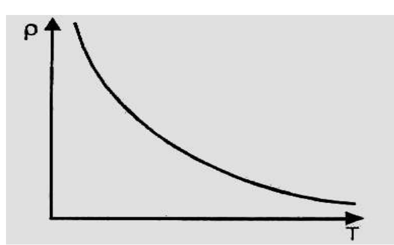
(b) Nichrome, which is an alloy of nickel and chromium, the resistivity is very large, but has a weak temperature dependence. It has residual resistivity even at absolute zero, whereas a pure metal has vanishing (or very small) resistivity at absolute zero.

(c) The resistivity of alloy manganin is nearly independent of temperature.

(ii) **For semiconductors and insulators:**

For insulators and semiconductors, it is not the relaxation time τ, but the number density, n of free charge carries that change more with temperature. With the increase in temperature, n increases as more number of electrons become free because of thermal agitation.



Variation of resistivity with temperature in semiconductor

Therefore, the resistivity of semiconductors and insulators decreases with increase in temperature.

CONDUCTANCE:

The reciprocal of resistance of conductor is called its conductance. It is denoted by G. Thus, the conductance of a conductor having resistance R is given by $G = \frac{1}{R}$.

SI unit of conductance is $ohm^{-1}(\Omega^{-1})$ or mho or Siemen (S).

Temperature dependence

Since $R \propto \rho$ and $G = \frac{1}{R}$, conductance of the material is inversely proportional to its resistivity.

So, the trend of variation of conductance of the conductor and semiconductor with temperature is just opposite of that of its resistivity.

CONDUCTIVITY:

The reciprocal of resistivity of conductor is called its conductivity. It is denoted by ρ.

Thus, $\sigma = \frac{1}{\rho}$, SI unit of conductivity is or $(\Omega - m)^{-1}$.

Temperature dependence

Conductivity, also, has the same trend of variation with temperature as that of conductance of a conductor or semiconductor.

Mobility: Drift velocity acquired by a charge under the effect of unit electric field applied is called its mobility.

$$\mu = \frac{V_d}{E}$$

PROBLEM RELATED TO OHM'S LAW, VARIATION OF RESISTANCE WITH TEMPERATURE:

SOLVED EXAMPLE:

Example.7 A wire has a resistance of 2.0Ω at 25ºC and 2.5Ω at 100 ºC. Find the temperature coefficient of resistance of the wire.

Solution. Using $R_1 = R_0(1+\alpha t)$, we get $2.5 = R_0(1+100\alpha)$

and $2.0 = R_0(1+25\alpha)$

$\therefore \frac{2.5}{2.0} = \frac{1+100\alpha}{1+25\alpha}$, solving, we get $\alpha = 3.6\times10^{-3} / ^{o}C$

Example. 8 Two copper wires of the same length have got different diameters.

(a) which wire has greater resistance ?

(b) greater specific resistance ?

Solution. (a) For a given wire,

$$R = \rho \frac{l}{A}$$

i.e., $R \propto \frac{1}{A}$

So, the thinner wire will have greater resistance.

(b) Specific resistance (ρ) is a material property. It does not depend on l or A.

So, both the wires will have same specific resistance.

Example.9 A wire has a resistance R. What will be its resistance if it is stretched to double its length?

Solution. Let V be the volume of wire, then V = Al,,

$$A = \frac{V}{l}$$

Substituting this in $R = \rho \frac{\ell}{A}$, we have

$$R = \rho \frac{l^2}{V}$$

So, for given volume and material (i.e., V and ρ are constant)

$$R \propto l^2$$

When l is doubled, resistance will become four times, or, the new resistance will be 4R.

Example.10 The number density of conduction electron in copper is 8.5 x 10^{28} (m^{-3}) and the mean free time τ between collisions is 2.5×10^{-14} s. What is the conductivity of copper?

Solution.

$$\sigma = \frac{ne^2\tau}{m}$$

$$= \frac{8.5 \times 10^{28} \times (1.6 \times 10^{-19})^2 \times 2.5 \times 10^{-14}}{9 \times 10^{-31}} = \frac{8.5 \times 1.6 \times 1.6 \times 2.5}{9} \times 10^7$$

$\approx 6 \times 10^7$ mho/m or Siemen/metre

Example.11 Two wires of the same material having lengths in the ratio of 1 : 2 and diameters in the ratio 2 : 1 are connected in series with a cell of emf 2 volt and internal resistance 1 ohm. What is the ratio of the potential difference across the two wires?

Solution. Since in series, same current flows, thus

$$\frac{V_1}{V_2} = \frac{IR_1}{IR_2} = \frac{R_1}{R_2}$$

$$= \frac{\ell_1}{\pi r_1^2} \times \frac{\pi r_2^2}{\ell_2} = \frac{\ell_1}{\ell_2} \times \frac{r_2^2}{r_1^2}$$

$$= \left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^2 = \frac{1}{8}$$

or $V_1 : V_2 = 1 : 8$

Example.12 If a copper wire is stretched to make its radius decrease by 0.15%, then the percentage increase in resistance is approximately

(1) 0.15% (2) 0.40%

(3) 0.60% (4) 0.90%

Solution. Due to stretching resistance changes are in the ratio $\frac{R_2}{R_1} = \left(\frac{r_1}{r_2}\right)^4$

or $R \propto r^{-4}$

or $\Delta R \propto -4r^{-5}\ \Delta r$

or $\frac{\Delta R}{R} = -4\frac{\Delta r}{r}$

$= 4 \times 0.15\%$ = 0.60% Ans. **(3)**

EXERCISE:

10. The product of resistivity and conductivity of a conductor is constant. Is this statement true or false ?

11. If E denotes electric field in a uniform conductor and V_d corresponding drift velocity of free electrons in the conductor, which of the following graphs are correct ?

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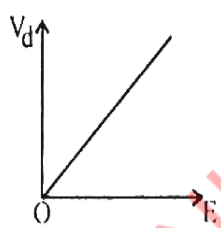

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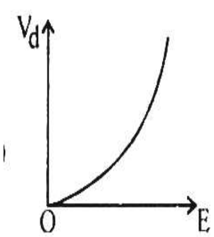

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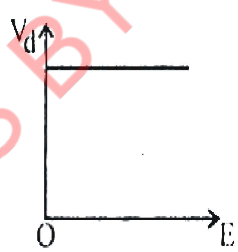

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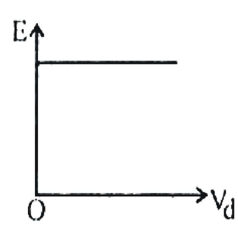
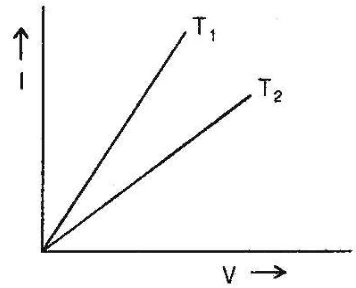


12. Two wires of same dimension but resistivities ρ_1 and ρ_2 are connected in series. The equivalent resistivity of the combination is

(1) $\rho_1 + \rho_2$ (2) $\frac{1}{2}(\rho_1 + \rho_2)$ (3) $\sqrt{\rho_1\rho_2}$ (4) $2(\rho_1 + \rho_2)$

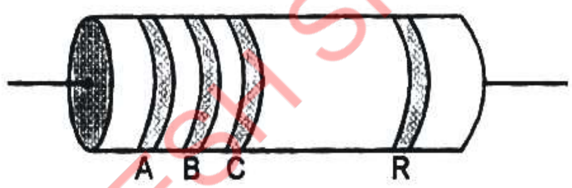
13. V-I graph for a metallic wire at two different temperature T_1 and T_2 is as shown in the following figure. Which of the two temperature higher and why ?



5. COLOUR CODE OF RESISTANCE:

Resistors are colour coded in order to indicate the resistance value and its percentage accuracy. The resistor has a set of co-axial coloured rings on it, with their significance as indicated in the table.

The first two bands from the end indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier and last band stands for the tolerance or possible variation in percent about the indicated value. If the fourth band is absent, it implies that the tolerance is $\pm 20\%$



COLOUR	LETTER AS AN AID TO MEMORY	NUMBER	MULTIPLIER	COLOUR	TOLERANCE
Black	B	0	10^0	Gold	5%
Brown	B	1	10^1	Silver	10%
Red	R	2	10^2	No Colour	20%
Orange	O	3	10^3		
Yellow	Y	4	10^4		
Green	G	5	10^5		
Blue	B	6	10^6		
Violet	V	7	10^7		
Grey	G	8	10^8		
White	W	9	10^9		
Gold			10^{-1}		
Silver			10^{-2}		

PROBLEM RELATED TO COLOUR CODING OF RESISTORS:

SOLVED EXAMPLES:

Example.13 For the given carbon resistor, let the first strip be yellow, second strip be red, third strip be orange and fourth be gold. What is its resistance ?

Solution. We know that the numbers for yellow, red, orange and gold are 4, 2, 3 and 5% respectively.

Hence the value of the given resistance $= 42 \times 10^3\ \Omega \pm 5\%$

Example.14 The resistance of the given carbon resistor is $2.4 \times 10^6\ \Omega \pm 5\%$ What are the sequence of colours on the strips provided on resistor ?

Solution. The value of the resistance of given carbon resistor,

$$R = 2.4 \times 10^6\ \Omega + 5\%$$
$$= 24 \times 10^5\ \Omega \pm 5\%$$

The colours attached to numbers 2, 4 and 5 are red, yellow and green. The colour for 5% is gold.

Therefore, the colours of strips in sequence for the given carbon resistor are red, yellow, green and gold.

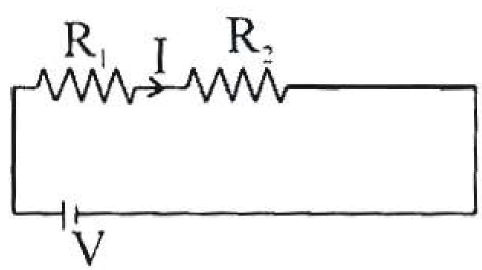
6. COMBINATION OF RESISTANCE:

Resistance in series:

Two or more resistors connected so that same current flow through each are siad to be connected in series.

$V_1 = IR_1, V_2 = IR_2$ and I = constant

Let the net potential difference be "V"



Then , $V = V_1 + V_2$ and $V = IR_{eqv.}$

$\Rightarrow IR_{eqv.} = IR_1 + IR_2$

$\therefore$ $\mathbf{R_{eqv.} = R_1 + R_2}$

$$V_1 = iR_1 = \frac{V}{R_1 + R_2} \times R_1,\quad V_2 = iR_2 = \frac{V}{R_1 + R_2} \times R_2$$

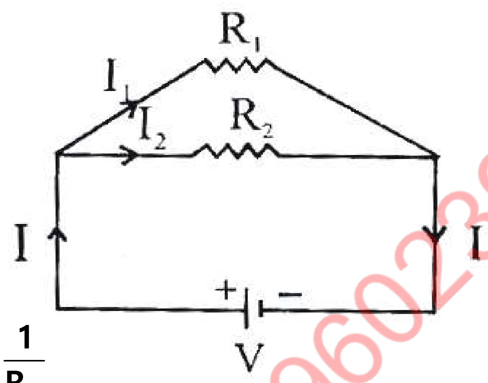
In series combination, voltage is distributed in the ratio of resistance

Resistance in parallel:

Two or more resistor connected as shown in the figure so that they have the same potential difference across them are said to be connected in parallel.

Each resistor has same potential difference but different current

$I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}$ and V = constant



Let the net current be I

Then, $I = \frac{V}{R_{eqv.}}$ and $I = I_1 + I_2$

$\Rightarrow \frac{V}{R_{eqv.}} = \frac{V}{R_1} + \frac{V}{R_2}$ $\therefore \frac{1}{R_{eqv.}} = \frac{1}{R_1} + \frac{1}{R_2}$

$V = i_1R_1 = i_2R_2 \Rightarrow \frac{i_1}{i_2} = \frac{R_2}{R_1}$

In parallel combination current is distributed in inverse ratio of resistance.

$i_1 = \left(\frac{i}{R_1 + R_2}\right) \times R_2$ and $i_2 = \left(\frac{i}{R_1 + R_2}\right) \times R_1$

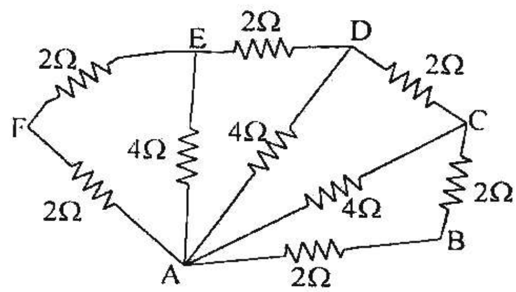
PROBLEM RELATED TO COMBINATIONS OF RESISTANCES:

SOLVED EXAMPLES:

Example.15 Find the effective resistance between points A and B in the figure shown..

Solution. The resistors AF and FE are in series and therefore their equivalent resistance is $2\Omega + 2\Omega = 4\Omega$. This in turn is connected in parallel with AE. Hence, their equivalent resistance between A and E is

$$\frac{4 \times 4}{4 + 4} = 2\Omega$$

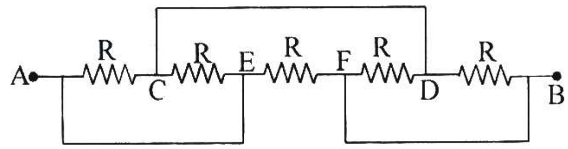


This 2Ω resistance between A and E is in series with ED and the combination is in parallel with AD. Their equivalent resistance between AD is again 2Ω.

Similarly, this 2Ω resistance between A and D is in series with DC and their combination is in parallel with AC. Their equivalent resistance between A and C is again 2Ω. The equivalent resistance between A and B is, therefore,

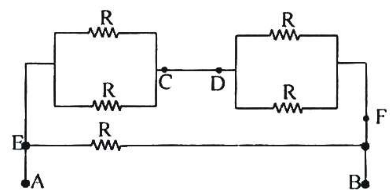
$$R_{eqv.} = \frac{4 \times 2}{4 + 2} = \frac{8}{6} = \frac{4}{3}\Omega$$

Example.16 Find equivalent resistance between points A and B as shown in figure.



Solution. From the figure, it is clear that $V_A = V_E$, $V_C = V_D$ and $V_F = V_B$

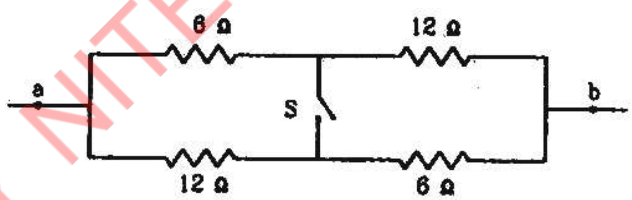
Therefore, circuit can be redrawn as shown



The equivalent resistance between A and C is R/2 and between C and B is R/2. These two resistances are in series and their equivalent is $R/2 + R/2 = R$.

This is in parallel with another resistance R. Hence, equivalent resistance between A and B is $\frac{R \times R}{R+R} = \frac{R}{2}$ohm

Example.17 Find the equivalent resistance of the network shown in figure between the points a and b when (a) the switch S is open (b) the switch S is closed.



Solution. (a) When the switch is open, $6\,\Omega$ and $12\,\Omega$ resistors on the upper line are in series giving an equivalent of $18\,\Omega$. Similarly, the resistors on the lower line have equivalent resistance $18\,\Omega$. These two $18\,\Omega$ resistances are connected in parallel between a and b so that the equivalent resistance is $9\,\Omega$.

(b) When the switch is closed, the $6\,\Omega$ and $12\,\Omega$ resistors on the left are in parallel giving an equivalent resistance of $4\,\Omega$. Similarly, the two resistors on the right half are equivalent to $4\,\Omega$. These two are connected in series between a and b so that the equivalent resistance is $8\,\Omega$.

EXERCISE:

14. The resistance of a wire is 10Ω. Its length is increased by 10% by stretching. The new resistance will now be

(1) 12Ω (2) 1.2Ω (3) 13Ω (4) 11Ω

15. Resistance of tungsten wire at 150°C is 133Ω. Its resistance temperature coefficient is 0.0045 /°C. The resistance of this wire at 500°C will be

(1) 180Ω (2) 225Ω (3) 258Ω (4) 317Ω

16. The colour code for a resistor of resistance $3.5k\Omega$ with 5% tolerance is

(1) Orange, green, red and gold (2) Red, yellow, black and gold

(3) Orange, green, orange and silver (4) Orange, green, red and silver

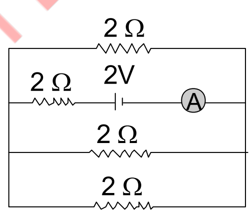
17. Masses of three wires of copper are in the ratio of 1 : 3 : 5 and their lengths are in the ratio of 5 : 3 : 1. The ratio of their electrical resistance are

(1) 1 : 3 : 5 (2) 5 : 3 : 1 (3) 1 : 15 : 125 (4) 125 : 15 : 1

18. If potential V = 100 $\pm$ 0.5 Volt and current I = 10 $\pm$ 0.2 amp are given to us, then what will be the value of resistance

(1) 10 $\pm$ 0.7 ohm (2) 5 $\pm$ 2 ohm (3) 0.1 $\pm$ 0.2 ohm (4) None of these

19. The reading of the ammeter as per figure shown is



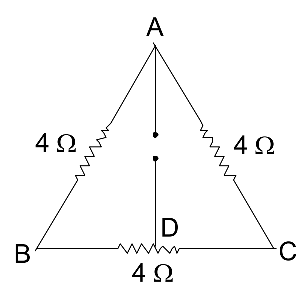
(1) $\frac{1}{8}A$

(2) $\frac{3}{4}A$

(3) $\frac{1}{2}A$

(4) 2 A

20. Three resistances of 4Ω each are connected as shown in figure. If the point D divides the resistance into two equal halves, the resistance between point A and D will be



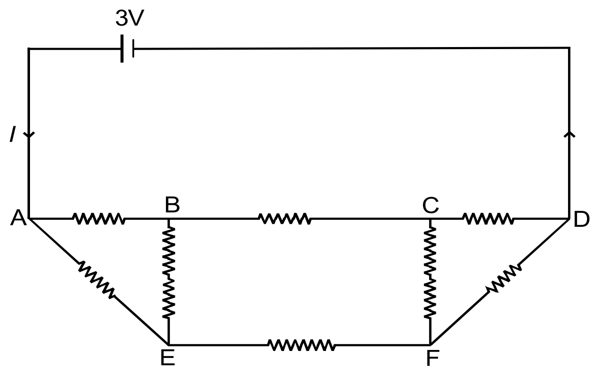
(1) 12Ω

(2) 6Ω

(3) 3Ω

(4) $1/3\Omega$

21. Figure shows a network of eight resistors, each equal to 2Ω, connected to a 3 V battery of negligible internal resistance. The current *I* in the circuit is

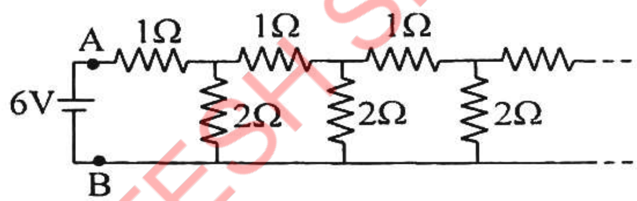


(1) 0.25 A (2) 0.50 A (3) 0.75 A (4) 1.0 A

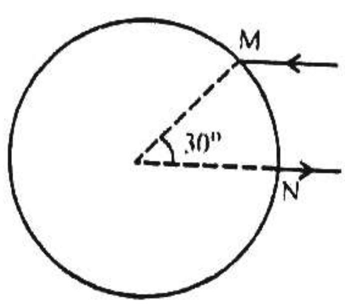
22. An infinite ladder is constructed with $1\ \Omega$ and $2\ \Omega$ resistors as shown in figure.

(a) Find the effective resistance between the points A and B

(b) Find the current that passes through the $2\ \Omega$ resistor nearest to the battery.



23. A uniform wire of resistance 36 ohm is bent in form of a circle. The effective resistance between points M and N is



(1) $2.75\ \Omega$ (2) $3.5\ \Omega$ (3) $33\ \Omega$ (4) $36\ \Omega$

7. SOURCE OF EMF:

It is a device which maintains a potential difference between two points in the circuit.

EMF (Electromotive force):

To have a steady current in a conductor, we need to have a supply of electric energy. A device which can supply energy to charged carriers and therby maintain their flow is called a source of electromotive force (emf). In the interior of source of emf, positive charges move from a point of low potential (negative terminal) to a point of higher potential (positive terminal). If ΔW is the work done by source, when ΔQ charge passes from lower potential (negative) to higher potential (positive) plate.

$$E = \frac{\Delta W}{\Delta Q}$$

Unit of emf = volt = joule / coulomb

The emf of a cell is the potential difference between the two electrodes in an open circuit, i.e, when no current is drawn from the cell.

The source of energy required for moving charges from negative to positive terminal may be chemical (as in a cell or a battery), mechanical (in a generator), thermal (in a thermopile) or radiation (in a solar cell).

The rate at which energy (power) is supplied by the emf is

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta Q}{\Delta t} E = EI$$

Internal Resistance (r) :

It is the resistance offered by the electrolyte of the cell to flow of charges (ions). The internal resistance can not be separated from the cell. It reduces the current that the emf can supply to the external circuit.

$$I = \frac{E}{R + r}$$

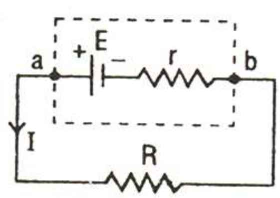
Terminal Potential Difference (V) :

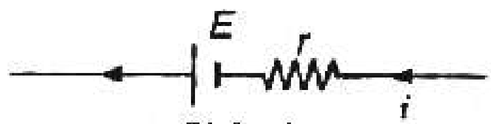
The potential difference between the two electrodes of a cell in a closed circuit (when current is drawn from the cell) is called terminal potential difference (V). The terminal potential difference between a and b

$$V = V_a - V_b,$$

$$V = IR = E - Ir$$

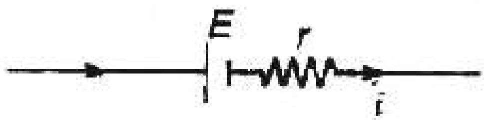
$$r = \frac{E - V}{I} = \frac{E - V}{V} \times R$$





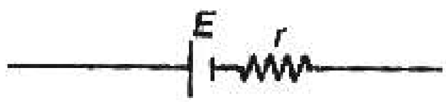
Discharging

$V = E - ir$ or $V < E$



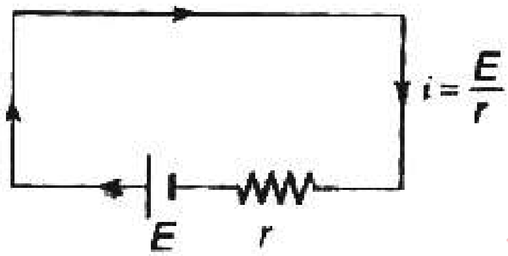
Charging

$V = E + ir$ or $V > E$



Open Circuit

$V = E$ if $i = 0$



Short Circuit

$V = 0$ if short circuited

PROBLEM RELATED TO EMF:

SOLVED EXAMPLES:

Example.18 A battery of emf 10 V and internal resistance 3 ohm is connected to a resistor. If the current in the circuit is 0.5 A, what is the resistance of resistor? What is the terminal voltage of the battery when the circuit is closed?

Solution. (i) $I = \frac{E}{R+r}$ or $0.5 = \frac{10}{3+R}$

Solve to get $R = 17\ \Omega$

(ii) $V = E - Ir = 10 - 0.5 \times 3$

$= 8.5$ Volt

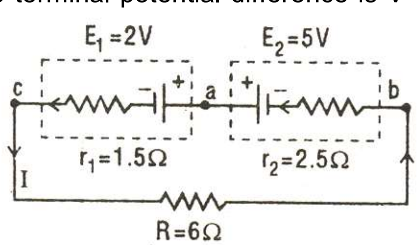
Example.19 See Fig., (i) What is the current in the circuit, (ii) What is the potential difference between points a and b, (iii) What is the potential difference between points a and c.

Solution. (i) $I = \frac{E_2 - E_1}{r_1 + r_2 + R} = \frac{5-2}{1.5+2.5+6} = \frac{3}{10} = 0.3\,A$

(When the cell is being discharged. The terminal potential difference is V=E–Ir)

$V_a - V_b = 5 - 0.3 \times 2.5$

$= 4.25$ Volt



Alternatively,

$V_a - V_b = E_1 + I(r_1 + R)$

$= 4.25$ Volt

(iii) Similarly, for $V_a - V_c$, go from a to E_1 to r_1 to c, then

$V_a - E_1 - Ir_1 = V_c$

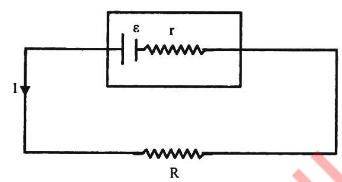
(when the cell is being charged, then the terminal potential difference V = E + Ir)

$V_a - V_c = E_1 + Ir_1 = 2 + 0.45$

$= 2.45$ volt

ENERGY, POWER AND HEATING EFFECT:

When a current I flows for time t from a source of emf E, then the amount of charge that flows in time t is Q = I t.



Electrical energy delivered W = QV = V I t

Thus, Power given to the circuit, = W/t = VI or V^2/R or I^2R

In the circuit, $EI = I^2R + I^2r$, where

E I is the rate at which chemical energy is converted to electrical energy, I^2R is power supplied to the external resistance R and I^2r is the power dissipated in the internal resistance of the battery.

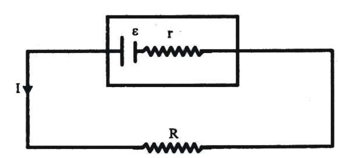
An electrical current flowing through conductor produces heat in it. This is known as Joule's effect. The heat developed in Joules is given by H = I^2.R.t

MAXIMUM POWER TRANSFER:

In a circuit, for what value of the external resistance the maximum power drawn from a battery? For the shown network power developed in resistance R equals

$$P = \frac{E^2.R}{(R+r)^2} \qquad \left(\because I = \frac{E}{R+r} \qquad and \qquad P = I^2R\right)$$

Now, for dP/dR = 0 (for P to be maximum $\frac{dP}{dR} = 0$)



$$\Rightarrow E^2.\frac{(R+r)^2 - 2(R)(R+r)}{(R+r)^4} = 0 \Rightarrow \qquad (R+r) = 2R \qquad \text{or } R = r$$

$\Rightarrow$ The power output is maximum, when the external resistance equals the internal resistance.

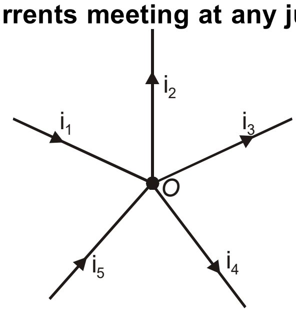
R = r

8. **KIRCHHOFF'S LAWS:**

(a) The junction law :

In an electric circuit, the algebraic sum of all the currents meeting at any junction is zero.

ie., $\Sigma i = 0$



From Kirchhoff's law for the point O,

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0$$

or $\quad i_1 + i_5 = i_2 + i_3 + i_4$

Sign convention:

currents entering the junction $\rightarrow (+)\text{ve}$

currents leaving the junction $\rightarrow (-)\text{ve}$

This law represents the law of conservation of charge, ie., when a constant current flows in a circuit, the charge does not accummulate at a junction or at any point of the circuit.

(b) Loop rule :

The algebraic sum of all the potential charges around any closed circuit is equal to zero

$$\Sigma iR = \Sigma \text{emf}$$

or $\quad \Sigma iR - \Sigma \text{emf} = 0$

Any potential drop is taken as negative and any potential increase is taken as positive. The net sum of all potential changes must be zero.

Note :

An increase in voltage is taken as negative (+)

A decrease in voltage is taken as positive (–)

HOW TO APPLY LOOP RULE IN A CIRCUIT

(1) If we traverse through a battery from "–ve" terminal to +ve terminal, there is rise in potential. So, change in potential is positive (+).

(2) If we traverse through a battery from +ve terminal to -ve terminal, there is fall in potential. So, change in potential is negative (-)

(3) If we traverse across a resistor in the direction of current, there is a fall in potential equal to iR. This is taken as negative.

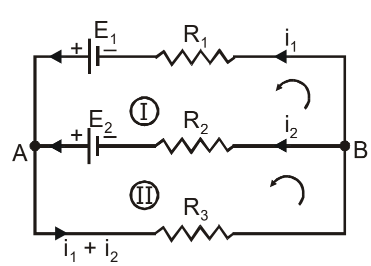
(4) If we traverse across a resistor in the direction opposite to current, there is a increase in potential equal to iR. This is taken as positive.

The above situations are shown in figure below.

$V_a - V_b = -\varepsilon \qquad V_a - V_b = +\varepsilon \qquad V_a - V_b = +iR \qquad V_a - V_b = -iR$

It is based on the principle of conservation of energy.

The loop law follows directly from the fact that electrostatic force is a conservative force and the work done by it in any closed path is zero.

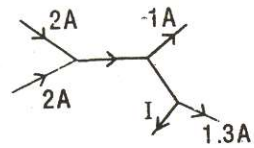


For (I) mesh : $i_1R_1 - i_2R_2 = E_1 - E_2$

For (II) mesh : $i_2R_2 + (i_1 + i_2)R_3 = E_2$

PROBLEM RELATED TO KIRCHHOFF'S LAW:

SOLVED EXAMPLES:



Example.20 Find the value of I in the circuit below :

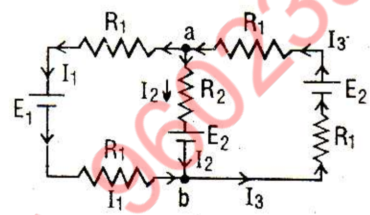
Solution. I = 4 – 1 – 1.3

= 1.7 A

Example.21 Figure shows a circuit whose elements have the following vaues : E_1 = 2V, E_2 = 6V, R_1 = 1.5 ohm and R_2 = 3.5 ohm. Find the curents in the three branches of the circuit.

Solution. From junction rule at **a**

$I_3 = I_1 + I_2$ .... (1)



For the left hand loop **aE_1ba,**

$$I_1R_1 - E_1 - I_1R_1 + E_2 + I_2R_2 = 0$$

or $E_2 - E_1 = 2I_1R_1 - I_2R_2$

or $4 = 3I_1 - 3.5I_2$ .... (2)

For the loop on right hand side starting from **a** (clockwise)

$$I_3R_1 - E_2 + I_3R_1 + E_2 + I_2R_2 = 0$$

$$2I_3R_1 + I_2R_2 = 0$$

Use $I_3 = I_1 + I_2$ Eq. (1)

$$2I_1R_1 + I_2(2R_1 + R_2) = 0$$

$3I_1 + 6.5I_2 = 0$ .... (3)

From (2), use $3I_1 = 4 + 3.5I_2$, in (3), to get $4 + 3.5I_2 + 6.5I_2 = 0$

or $I_2 = -0.4\,\text{Amp}$

Substitute it in (2), to get

$I_1 = 0.87\,\text{Amp}$

Therefore, $I_3 = I_1 + I_2 = 0.87 + (-0.4)$

$= 0.47\,\text{Amp}$

Example.22 What is the potential difference between points **a** and **b** in the circuit of above figure.

Solution. In going from a (potential V_a) to b (potential V_b), we have

$$V_a - I_2R_2 - E_2 = V_b$$

$$V_a - V_b = E_2 + I_2R_2$$

$$= 6 + (-0.4) \times (3.5)$$

$$= 6 - 1.4 = 4.6\,\text{volt}$$

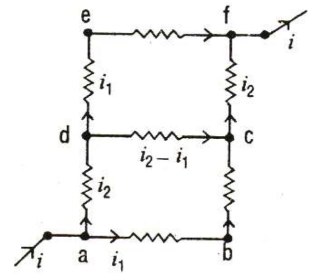
Example.23 The current in a,b for the circuit given here is (each resistance = R ohm).

(1) i/5

(2) 2i/5

(3) 3i/5

(4) i



Solution. From symmetry the currents in various branches are as shown. Now for path abc, $V_a - V_c = 2Ri_1$, and for path adc, $V_a - V_c = 2Ri_2 - Ri_1$. Therefore,

$$2Ri_1 = 2Ri_2 - Ri_1$$

or $3i_1 = 2i_2$

Further $i_1 + i_2 = i$. Thus $i_1 + \left(\frac{3}{2}\right)i_1 = i$ or $i_1 = \left(\frac{2}{5}\right)i$. The answer is (2) 2/5 i irrespective of the value of R]

Example.24 In a circuit shown in fig E, F, G and H are cells of emf 2, 1, 3 and 1 V respectively and their internal resistance are 2, 1, 3 and 1Ω respectively. Calculate

(i) The potential difference between B and D

(ii) The potential difference across the terminals of the cells G and H.

Solution. Considering mesh DBAD

$$-2i_2 + 2i_1 + i_1 = 2 - 1$$

$\Rightarrow$ $3i_1 - 2i_2 = 1$

Considering mesh DCBD,

$$3(i_1 + i_2) + (i_1 + i_2) + 2i_2 = 3 - 1 \qquad \text{......... (i)}$$

$\Rightarrow$ $4i_1 + 6i_2 = 2$

Solving (i) and (ii) $i_1 = \frac{5}{13}A$ and $i_2 = \frac{1}{13}A$

$\therefore$ $i_1 + i_2 = \frac{6}{13}A$

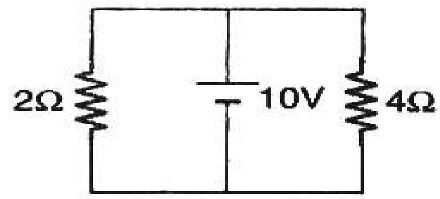
$\therefore$ Pd across BD, $V_{BD} = i_2R = \frac{2}{13}V$ ...... (ii)

Pd across cell G, $V_{CD} = 3 - (i_1 + i_2) = 3 - \frac{6}{13} = \frac{33}{13}V$

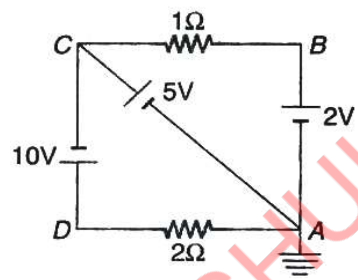
Pd across cell H, $V_{CB} = 1 - (i_1 + i_2) = \frac{9}{13}V$

EXERCISE:

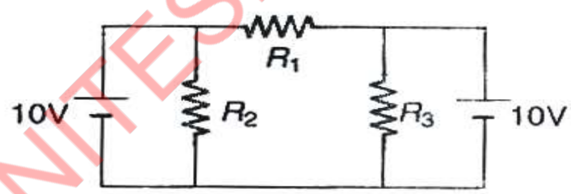
24. Find the current through 2Ω and 4Ω resistance.



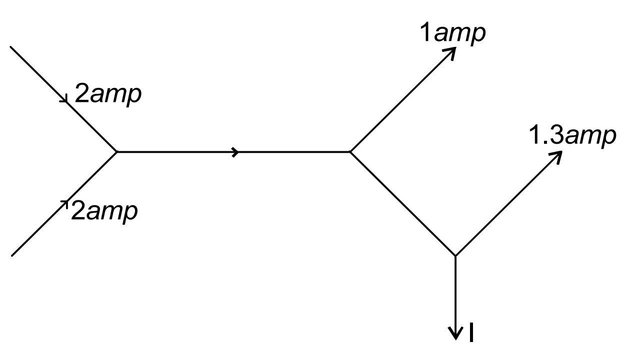
25. In the circuit shown in figure, find the potentials of A, B, C and D and the current through 1Ω and 2Ω resistance.



26. In the circuit shown in figure, $R_1 = R_2 = R_3 = 10\Omega$. Find the currents through R_1 and R_2.

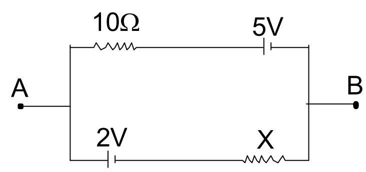


27. The figure below shows currents in a part of electric circuit. The current i is



(1) 1.7 amp (2) 3.7 amp (3) 1.3 amp (4) 1 amp

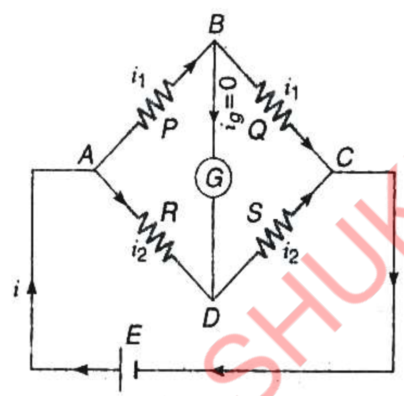
28. Two resistances R_1 and R_2 are joined as shown in the figure to two batteries of e.m.f. E_1 and E_2. If E_2 is short-circuited, the current through R_1 is



(1) 5Ω (2) 10Ω (3) 15Ω (4) 20Ω

9. WHEATSTONE'S BRIDGE:

It is an arrangement of four resistances used for measuring one of them in terms of the other three.



When some potential difference is applied between A and C, the potential difference between B and D depends only on the ratio $\frac{P}{Q}$ and $\frac{R}{S}$.

If $\frac{P}{Q}=\frac{R}{S}$, no current flows through the galvanometer and Wheatstone bridge is said to be balanced.

Since "Wheatstone Bridge" method is a "NULL method", the results are not affected by internal resistances of cells and resistances of ammeters and voltmeters.

PROOF:

At balance point, $i_g = 0,\ V_B = V_D$

Under this condition $V_A - V_B = V_A - V_D$

or $i_1P = i_2R$

or $\frac{i_1}{i_2}=\frac{R}{P}$ .....(i)

Similarly, $V_B - V_C = V_D - V_C$

or $i_1Q = i_2S$

or $$\frac{i_1}{i_2} = \frac{S}{Q} \quad ....(ii)$$

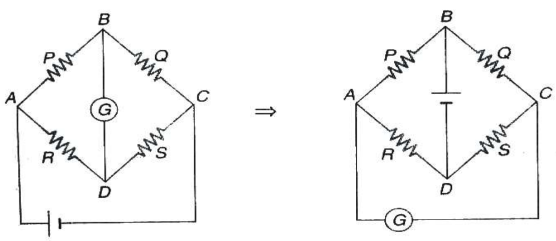
From Eqs. (i) and (ii),

$$\frac{R}{P} = \frac{S}{Q}$$

or $$\frac{P}{Q} = \frac{R}{S}$$

IMPORTANT POINTS:

(i). In Wheatstone bridge, cell and galvanometer arms are interchangable.



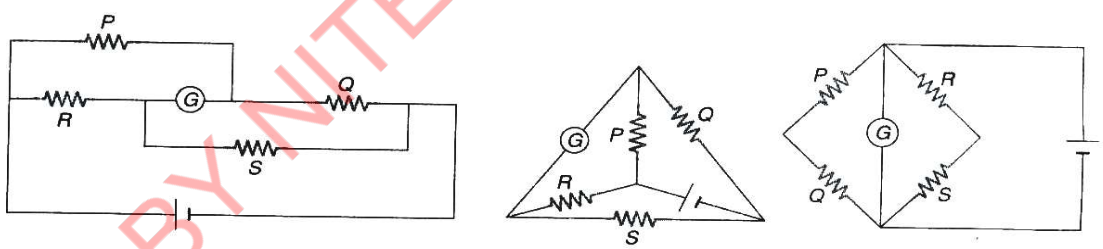
In both the cases, condition of balanced bridge is

$$\frac{P}{Q} = \frac{R}{S}$$

(ii) If bridge is not balanced current will flow from D to B if,

$PS > RQ$

(iii) Following are given few circuits which are basically Wheatstone's bridge circuits.



PROBLEM RELATED TO WHEATSTONE'S BRIDGE:

SOLVED EXAMPLES:

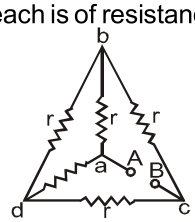
Example 25. In the adjoining network of resistors, each is of resistance r ohm, the equivalent resistance between points A and B is

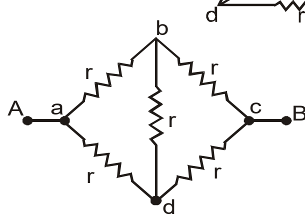
(1) 5r

(2) 2r/3

(3) r

(4) r/2

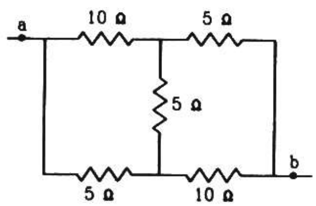


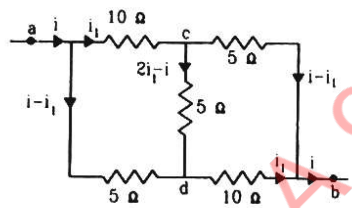


Solution. Imagine, Aa being pulled on the left side, then abcd becomes a balanced Wheatstone bridge (figure). The arm bd can be ignored. Then resistance between A, B becomes = r. The answer is (3).

Example.26 Find the equivalent resistance between the points a and b of the circuit shown in figure.

Solution. Suppose a current i enters the circuit at the point a, a part i_1 goes through the 10Ω resistor and the rest $i - i_1$ through the 5Ω resistor. By symmetry, the current i coming out from the point b will be composed of a part i_1 from the 10Ω resistor and $i - i_1$ from the 5Ω resistor. Applying Kirchhoff's law, we can find the current through the middle 5Ω resistor. The current distribution is shown in figure





$$V_a - V_b = (V_a - V_c) + (V_c - V_b)$$
$$= (10\Omega)i_1 + (5\Omega)(i - i_1)$$
$$= (5\Omega)i + (5\Omega)i_1. \qquad ....(i)$$

Also,
$$V_a - V_b = (V_a - V_c) + (V_c - V_d) + (V_d - V_b)$$
$$= (10\Omega)i_1 + (5\Omega)(2i_1 - i) + (10\Omega)i_1$$
$$= -(5\Omega)i + (30\Omega)i_1 \qquad ....(ii)$$

Multiplying (i) by 6 and subtracting (ii) from it, we eliminate i_1 and get,

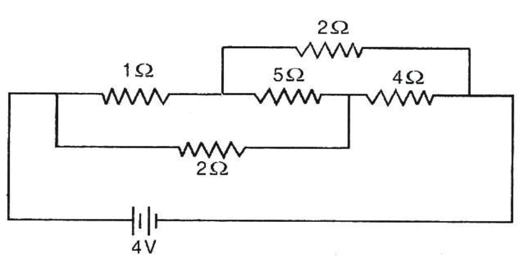
$$5(V_a - V_b) = (35\Omega)i$$

or
$$\frac{V_a - V_b}{i} = 7\Omega$$

Thus, the equivalent resistance between the points a and b is 7Ω

Example.27 Calculate the current drawn from the battery in the given network sketched here.

Solution.

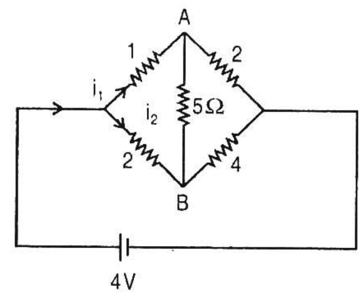


Given circuit reduces to a balanced Wheatstone bridge.

$\Rightarrow 4 = i_1 + 2i_1$ or $i_1 = \frac{4}{3}$

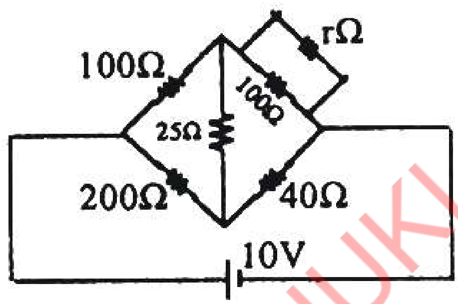
Also $4 = 2i_2 + 4i_2$ or $i_2 = \frac{2}{3}$

$\Rightarrow i = i_1 + i_2 = 2A$

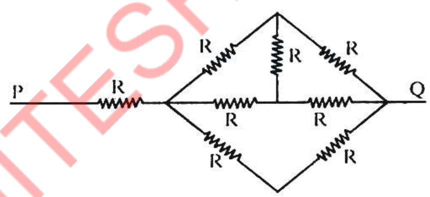


EXERCISE:

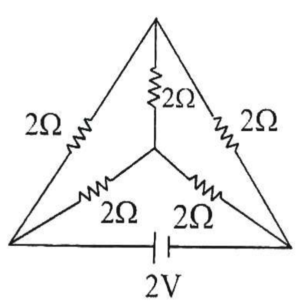
29. In the circuit given below, there is no current through 25Ω element. What is the value of r ?



30. Find the effective resistance between P and Q shown in figure.



31. Find the current through the cell, in the figure shown.

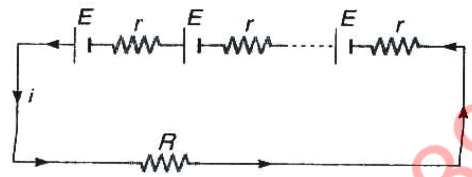


10. GROUPING OF CELLS:

Cells are usually grouped in following three ways.

1. Series Grouping:

Suppose n cells each of emf E and internal resistance r are connected in series as shown in figure.



Then, Net emf = nE

Total internal resistance = nr

Total resistance = nr + R

$\therefore$ Current in the circuit $i = \dfrac{\text{net emf}}{\text{total resistance}}$

or $i = \dfrac{nE}{nr + R}$

Note: If polarity of m cells is reversed, then equivalent emf = (n - 2m) E. While total resistance is still nr + R

$$\therefore \quad i = \frac{(n - 2m)E}{nr + R}$$

Special cases:

(a) If r is negligible, then

$I = \dfrac{nE}{R}$ = n x current obtained from one cell.

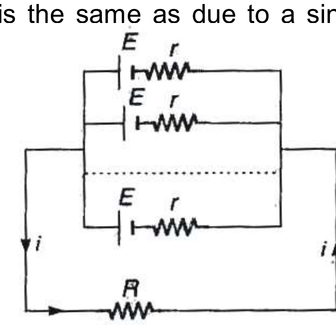
When the internal resistance is negligible, the current becomes n times the current due to one cell.

(b) R < < nr

$$I = \frac{E}{r}$$

Thus current in external resistance is the same as due to a single cell.

2. Parallel grouping:



Net emf = E

Total internal resistance = $\dfrac{r}{n}$

Total resistance = R + $\dfrac{r}{n}$

$$i = \frac{E}{R + r/n}$$

Special cases:

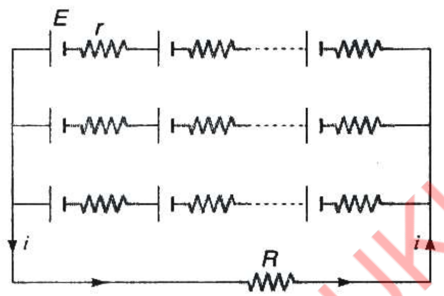
(a) If $\frac{r}{n} << R,\ i = \frac{E}{R}$

Thus current in external resistance is the same as due to a single cell.

(b) If $\frac{r}{n} >> R,\ i = \frac{nE}{r}$

When the external resistance is negligible, the current becomes n times the current due to one cell.

3. Mixed Grouping:



There are n identical cells in a row and number of rows are m. Emf of each cell is E and internal resistance is r. Treating each row as a single cell of emf nE and internal resistance nr,

we have

Net emf = nE

Total internal resistance $= \frac{nr}{m}$

Total external resistance = R

$\therefore$ Current through the external resistance R is,

$$i = \frac{nE}{R + \frac{nr}{m}}$$

a. Current in the circuit will be maximum when

$$\frac{R}{n} = \frac{r}{m}$$

or $R = \frac{nr}{m}$, Total external resistance = Total internal resistance

So $I_{max} = \frac{nE}{2R} = \frac{mE}{2r}$

b. Power transferred to the load will be maximum when $\frac{R}{n} = \frac{r}{m}$ $\left[since \quad P = \frac{E^2R}{\left(\frac{R}{n} + \frac{r}{m}\right)^2} \right]$

$$P_{max} = \frac{n^2E^2}{4R} = \frac{m^2E^2}{4r^2}R$$

PROBLEM RELATED TO GROUPING OF CELL:

SOLVED EXAMPLES:

Example.28 Twelve cells each having the same emf and negligible internal resistanc;e are kept in a closed box. Some of the cells are connected in the reverse order. This battery is connected in series with an ammeter, an external resistance R and two cells of the same type as in the box. The current when they aid the battery is 3 ampere and when they oppose, it is 2 ampere. How many cells in the battery are connected in reverse order?

Solution. Let n cells are connected in reverse order. Then emf of the battery is

$$E' = (12 - n)E - nE$$

$$= (12 - 2n)E$$

In case (i)

$$I = \frac{E' + 2E}{R} = 3$$

or $E' + 2E = 3R$,

or $(14 - 2n)\ E = 3R$ .... (1)

In case (ii)

$$I = \frac{E' - 2E}{R} = 2$$

or $E' - 2E = 2R$,

or $(10 - 2n)\ E = 2R$ .... (2)

Dividing (1) and (2)

$$\frac{14 - 2n}{10 - 2n} = \frac{3}{2}$$

or n = 1

One cell is connected in reverse order.

Example.29 8 cells are grouped to obtained the maximum current through a resistance of 2 ohm. If the emf of each cell is 2 volt and internal resistance is 1 ohm. Grouping of cells will have :

Solution. Let number of rows = m, Number of cells in a row = n

$$m \times n = 8 \quad ....(i)$$

For maximum current $R = \frac{nr}{m}$ ....(ii)

from eq. (i) and (ii) $m = 2, \quad n = 4$

Answer will be two rows of four cells

Example.30 A battery of 24 cells, each of emf 1.4 V and internal resistance 2Ω is to be connected so as to send maximum current through an external resistor of 12Ω resistance. (i) How are they to be connected ? (ii) Find the strength of current in each of the cells (iii) The potential difference across the external resistance.

Solution. (i) Let m be the number of cells in each row and let there be n rows in parallel, Then the total number of cells = nm = 24. Resistance of each row = $2m\Omega$.

There are n such rows in parallel. Therefore the total internal resistance of the battery = $2m\Omega$.

For the current in the external resistance to be maximum, the necessary condition is that the internal resistance of the battery must be equal to the external resistance.

$\Rightarrow \quad 2m/n = 12$

But mn = 24

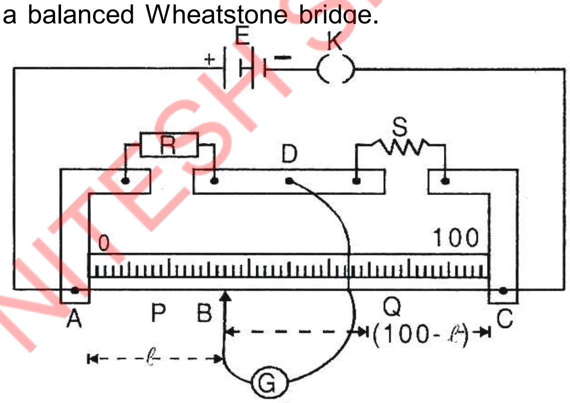
$\Rightarrow \quad 2m/n = 12$ and n = 2

Thus there must be two rows, each containing 12 cells in series.

11. METRE BRIDGE:

A meter-bridge is the simplest special case of a Wheatstone bridge, used to compare resistances very accurately.

In the figure, AC is a wire of uniform material and cross section clamped between two metallic strips bent at right angles. A known resistance S is connected between one gap and the resistance R connected between the other gap is to be measured. A galvanometer is connected using a jockey across BD. The end B is free to slide along the meter strip AB. AC is connected by a battery or cell. To find the resistance R, the slider B is moved along the wire till the null point is reached. Null point on the meter wire is the location of slider B from from end A for which the galvanometer shows null reading. For suitable values, R, S, P and Q can form a balanced Wheatstone bridge.



Then, $\frac{R}{S} = \frac{P}{Q}$

Where, $P = \frac{\rho \times \ell}{A}$ and $Q = \frac{\rho \times (100 - \ell)}{A}$ and S is a known resistance.

$\Rightarrow \quad \frac{R}{S} = \frac{\ell}{100 - \ell}$ .... (1)

In the experimental set-up the construction allows us to vary P and Q and thus achieve the balanced wheatstone bridge condition. The comparison of resistances is done by using the above equation (1). To find the resistivity of the unknown resistance R, we have to measure the length and cross section area of the wire and use the formula $R = \frac{\rho \times \ell}{A}$, where, ρ is the resistivity.

PROBLEMS RELATED TO METRE BRIDGE:

SOLVED EXAMPLES:

Example.31 In a metrebridge experiment a resistor of resistance R is kept in the left gap in all the observations. When resistance R_1 and R_2 are connected in turn in the right gap, the balance point is obtained at 60 cm and 50 cm from zero end of the wire. Find the position of the balance point, when the right gap contains R_1 and R_2 (a) in series and (b) in parallel.

Solution. When resistance R_1 is in the right gap, the balancing length of bridge wire is 60 cm.

Then $$\frac{R}{R_1} = \frac{60}{100-60} = \frac{60}{40}$$

or $$R_1 = \frac{40}{60}R = \frac{2}{3}R.$$

When resistance R_2 is in the right gap, the balancing length of bridge wire is 50 cm. Then

$$\frac{R}{R_2} = \frac{50}{100-50} = 1 \quad \text{or} \quad R_2 = R$$

(a) When R_1 and R_2 are in series then total resistance in right gap

$$= R_1 + R_2 = \frac{2}{3} + R = \frac{5}{3}R.$$

If the balancing length is l, then

$$\frac{R}{(5R/3)} = \frac{l}{(100-l)} \quad \text{or} \quad \frac{3}{5} = \frac{l}{(100-l)}$$

On solving, $l = 37.5\,\text{cm}$

(b) When R_1 and R_2 are in parallel, then the total resistance in right gap

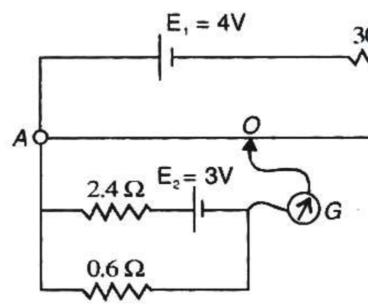
$$R_1 = \frac{R_1R_2}{R_1+R_2} = \frac{\frac{2R}{3} \times R}{\frac{2R}{3} + R} = \frac{2}{5}R.$$

If the balancing length is l, then

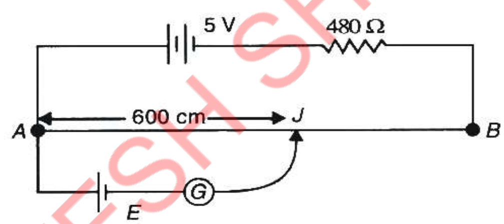
$$\frac{R}{2R/5} = \frac{l}{(100-l)} \quad or \quad \frac{5}{2} = \frac{l}{(100-l)}$$

On solving, $l = 71.43\,\text{cm}$

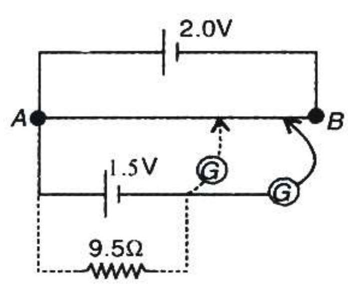
33. AB is 2 m long uniform wire of 20Ω resistance. The other data are as shown in the circuit diagram given below: Calculate (i) potential gradient along AB, and (ii) length AO of the wire, when the galvanometer shows no deflection. (i) 0.8 V/m (ii) 0.6 V; AO = 75 cm



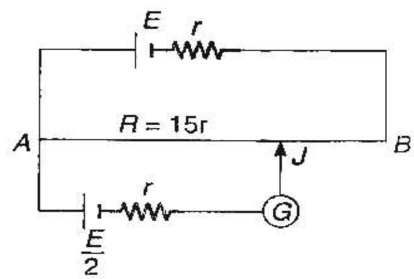
34. A 10 m long wire AB of uniform area of cross-section and 20Ω resistance is used as a potentiometer wire. This wire is connected in series with a battery of 5 V and a resistor of 480Ω. An unknown e.m.f. is balanced at 600 cm of the wire as shown in the figure Calculate:
(i) the potential gradient for the potentiometer wire.
(ii) the value of unknown e.m.f. E.



35. Figure shows a 2.0 V potentiometer used for the determination of internal resistance of a 1.5 V cell. The balance point of the cell in open circuit is 76.3 cm. When a resistor of 9.5Ω is used in the external circuit of the cell, the balance point shifts to 64.8 cm length of the potentiometer wire. Determine the internal resistanceof the cell.



36. The potentiometer wire AB is 600 cm long.
(a) At what distance from A should the jockey J touch the wire to get zero deflection in the galvanometer.
(b) If the jockey touches the wire at a distance 560 cm from A, what will be the current through the galvanometer.



13. GALVANOMETER:

It is a coil suspended between the poles of a magnet. When a current is passed through it, it deflects. The angle of deflection is proportional to the current going through it. A needle is fixed to the coil. When the coil deflects, the needle moves on a graduated scale. When no current passes through the coil, the needle stays in the middle of the graduated scale, marked zero. The current can be passed in either direction and needle deflects accordingly towards left or right.

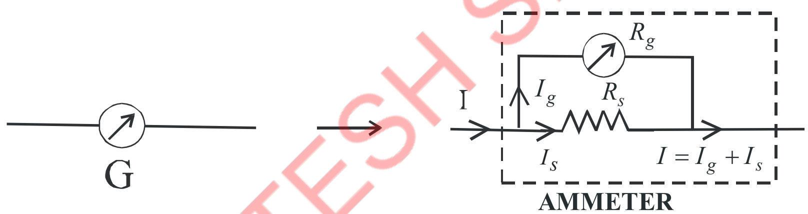
CONVERSION OF GALVANOMETER:

GALVANOMETER INTO AMMETER:

Ammeter : The instrument used for measuring current in circuit is called ammeter. The moving coil galvanometer is not very suitable for measuring current in a branch of a circuit for following two reasons.

(1) It is a very sensitive device and the coil gives full scale deflection for a very small value of current.

(2) Any device which measures current is connected in series in a branch through which current is to be measured. Hence resistance of the current measuring device should be very low, otherwise it will reduce the current flowing in the branch. The moving coil galvanometer is not suitable because it has fairly high resistance. (10 – 1000 Ω).



Ammeter: A low resistance called shunt connected in parallel with the galvanometer is called Ammeter.

If I_g is the galvanometer current for full scale deflection, and I the required range of ammeter then

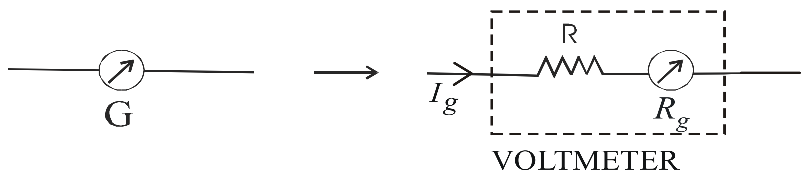
$$(I - I_g)R_s = I_g R_g \quad \text{or} \quad R_s = \frac{I_g}{I - I_g} R_g$$

where R_s and R_g are the resistances of the shunt and galvanometer coil respectively. The resistance of the ammeter so constructed is

$$R_A = \frac{R_g R_s}{R_g + R_s} = R_s \left[\frac{1}{1 + R_s / R_g} \right] < R_s$$

GALVANOMETER INTO VOLTMETER:

Voltmeter: Voltmeter is used for measuring potential difference between two points in a circuit. It is connected in parallel with that part of the circuit across which potential difference is to measured. The voltmeter must have a very high resistance, otherwise it will change the current in the circuit and thereby the potential difference across the part of the circuit which it is measuring.



The measured potential difference will be less than the actual.

A voltmeter is made by connecting a very high resistance R in series with the moving coil galvanometer.

If V is the desired range of the voltmeter and R the additional series resistance then

$$V = I_g(R + R_g)$$

or $$R = \frac{V}{I_g} - R_g$$

and $$R_V = R + R_g$$

PROBLEM RELATED TO VOLTMETER AND AMMETER:

SOLVED EXAMPLES:

Example.38 A galvanometer has a coil of resistance 200Ω showing a full scale deflection at $80\mu A$. What resistance should be added to use it as (a) a voltmeter of range 100V (b) an ammeter of range 10mA?

Solution. (a) When a current of $80\mu A$ flews through the galvanometer (full scale deflection) the potential difference of 100V should appear across the terminals of the voltmeter. We add a resistance R in series to achieve this.

$$80\mu A \times (200+R) = 100$$

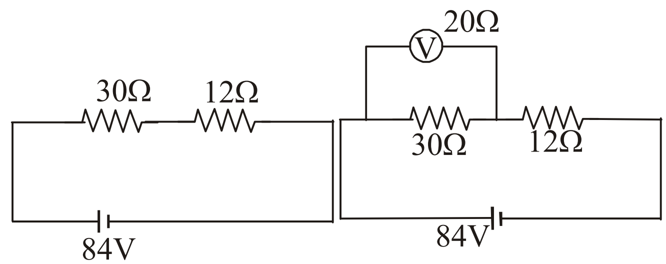
or $$R = 1.25\times10^6 - 200 \cong 1.25\times10^6\,\Omega$$

(b) When a current of 10mA flows through the ammeter, $80\mu A$ current should flow through the galvanometer coil. A resistance r in parallel to the coil is attached to achieve this.

$$80\mu A = (10mA)\frac{r}{r+200} \qquad \text{or,} \qquad r \cong 1.6\Omega$$

Example.39 (a) Find the potential drop across two resistors shown in the figure.

(b) A voltmeter of resistance 20Ω is used to measure the potential difference across 30Ω resistor. What is the reading of the voltmeter ?



Solution. (a) The current in the circuit is $i = \frac{84}{30+12} = 2A$

The potential difference across 30Ω resistor is 30 × 2 = 60V

and the potential drop across 12Ω resistor is 12 × 2 = 24V

(b) The equivalent resistance, when voltmeter is connected across 30Ω resistor is

$$R = 12 + \frac{20 \times 30}{20+30} = 24\Omega,$$

Therefore, the current through the battery is $i = \frac{84}{24} = 3.5A$

The effective resistance across the points between which voltmeter is connected is

$R_{eff} = \frac{30 \times 20}{30+20} = 12\Omega$, Therefore, the potential drop across the voltmeter is $i \times R_{eff} = 3.5 \times 12 = 42V$ and hence the reading of the voltmeter is 42V.

EXERCISE:

37. The range of a voltmeter of resistance 300Ω is 5 V. Find the resistance to be connected to convert it into Ammeter of range 5 Amp.

38. A milliammeter of range 10 mA has a coil of resistance 1Ω. How it will be converted into

(a) Ammeter of range 1 Amp.

(b) Voltmeter of range 10 V.